Landau levels

Consider motion in a strong magnetic field

$\omega_c \tau \gg 1$ (regime of „Quantum Hall" effect)

$\omega_c = \frac{|e|B}{mc}$ – cyclotron frequency

The Hamiltotian of the system

$$\hat{H} = \frac{1}{2m}\left(\hat{\vec{p}} - \frac{e}{c}\vec{A}\right)^2$$

Pick the gauge:

$\vec{A} = (0, Hx, 0)$ – Landau gauge

($\vec{A} = (-Hy, 0, 0)$ – also Landau gauge)

The Ham-n then is translationally invariant in the y direction

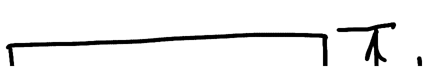
$$\psi = e^{ip_y y}\chi(x)$$

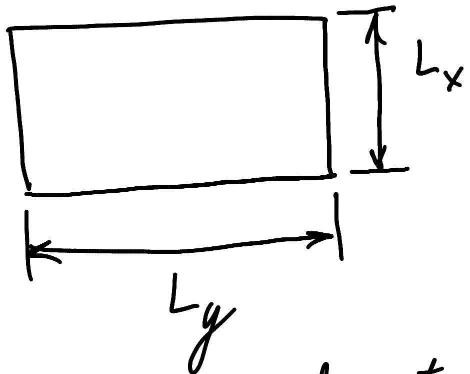
$$\left[-\frac{1}{2m}\partial_x^2 + \frac{1}{2m}\left(p_y - \frac{e}{c}Bx\right)^2\right]\chi = E\chi$$

Change variables: $p_y - \frac{e}{c}Bx \to -\frac{e}{c}B\tilde{x}$

$\tilde{x} = x - \frac{c}{eB}p_y$

To determine in what ranges all variables may change, consider a system of specific shape





$\tilde{x}$ varies on characteristic scales shorter than the sizes of the system

x may change from $-\frac{L_x}{2}$ to $+\frac{L_x}{2}$

Then p_y will vary between $-\frac{eB}{2c}L_x$ and $+\frac{eB}{2c}L_x$

The total number of values of p_y is

$$\frac{|e|B}{2\pi c} L_x L_y \equiv \gamma = \frac{|e|B}{2\pi c} S = \frac{BS}{\varphi_0}$$

The Ham-n after the change of variables $x \to \tilde{x}$:

$$\left[-\frac{1}{2m}\partial_{\tilde{x}}^2 + \frac{e^2B^2}{2mc^2}\tilde{x}^2\right]\chi = E\chi$$

The Ham-n is p_y independent, but all levels acquire an extra degeneracy of γ

$$\left(-\frac{1}{2m}\partial_{\tilde{x}}^2 + \frac{m\omega_c^2}{2}\tilde{x}^2\right)\chi = E\chi$$

– the Ham-n of a harmonic oscillator

$$\boxed{E_n = \hbar\omega_c\left(n+\frac{1}{2}\right)}$$ – Landau levels

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each has degeneracy $\gamma = \frac{BS}{\varphi_0}$

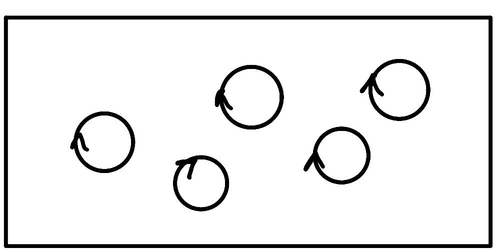
A characteristic lengthscale

$$\boxed{l_B = \sqrt{\frac{\hbar}{m\omega_c}} = \sqrt{\frac{\hbar c}{|e|B}}}$$

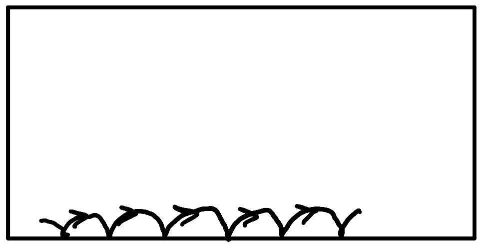
The states in the bulk of the system carry no current

$$\frac{\partial E_n}{\partial p_x} = \frac{\partial E_n}{\partial p_y} = 0$$

May be expected to be an insulator; interplay with disorder will be important though



The edge states are different though...



An effectively 1D conducting channel

The energy of a state vs. mean distance to the wall